THE CONCEPT OF REPRESENTATIVE DIRECTIONS

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The simulation of industrial components with regard to their mechanical behaviour is basically achieved by means of the finite element method. Thus, for any particular material class a constitutive model is needed, that is able to predict the complete correlation between the given strain condition and the corresponding stress condition, while also considering the loading history. In general, this is equivalent to the input of six independent coefficients of the strain tensor to the user subroutine and the output of six stress values of the corresponding stress tensor. The development of such a three-dimensional constitutive model usually takes a lot of time but often leads to an intermediate stage, that at least enables the prediction of the material behaviour for uniaxial tension. In such a situation it would be a great advantage to find a sufficient approximation of the prospective general behaviour of a constitutive model by considering the uniaxial material behaviour only. This is the idea of the so-called concept of representative directions.

The concept of representative directions developed by Ihlemann [1] is intended to generalize one-dimensional material models for uniaxial tension to fully three-dimensional constitutive models for the finite element method. The concept is already implemented into the finite element programs ABAQUS and MSC.MARC. The theory of the concept deals with a selection of evenly distributed directions in space. According to the external deformation of each material point the resulting elongations along those representative directions can be calculated by using the right Cauchy-Green tensor. For each of these elongations the corresponding uniaxial stress response is identified by the one-dimensional material model. The demand, that all the uniaxial stress responses achieve the same stress power as the overall stress tensor concerning the actual deformation, finally leads to an equation for calculating the demanded second Piola-Kirchhoff stress tensor.

A uniform distribution of representative directions minimizes the dependence of the material behaviour on the orientation of those directions (anisotropy). The generation of evenly distributed directions in space is achieved by simulating repelling electric charges bonded to the surface of the sphere, whereas the unit vectors to the points in their equilibrium state can be used as representative directions. Finally, each direction is scaled with the surface area of an associated Voronoi cell. For a further reduction of anisotropy, the uniaxial stress responses should be calculated with those elongations, that are averaged over several elongations within that Voronoi cell.

The implementation of the concept enables finite element simulations of inhomogeneous stress conditions, though the input model predicts uniaxial material behaviour only. The calculation of the stiffness matrix is achieved by differentiating the second Piola-Kirchhoff stress tensor with respect to the right Cauchy-Green tensor almost completely analytically. The only elements of the generalized material model differentiated numerically are the uniaxial stress responses along the representative directions. The advantage over a complete numerical differentiation of the whole stress tensor is a considerable reduction of the simulation runtime.

[1] Ihlemann, J.: Generalization of one-dimensional constitutive models with the concept of representative directions, in: A. Boukamel, L. Laiarinandrasana, S. Méo & E. V (Editors): Constitutive Models for Rubber V, Taylor & Francis, S. 29-34. London: 2007

[2] Ihlemann, J.: Kontinuumsmechanische Nachbildung hochbelasteter technischer Gummiwerkstoffe. Dissertation Universität Hannover. Düsseldorf: VDI-Verlag 2003

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