

Novel Approaches to the Analysis of Localised  
Stress Concentrations in Deformed Elastomers

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## **Abstract**

The research described here is concerned with improving finite element analysis (FEA) of rubber components subjected to dynamic loading, particularly in respect of accurate modelling of stress softening using standard software codes and phenomenological material models. The research required the design and implementation of a user subroutine capable of inclusion in standard highly nonlinear codes. Experimental methods were employed to corroborate the FEA simulations and so validate the subroutine. This was also achieved using 3D image correlation photogrammetry (ICP) and other novel experimental procedures. From this experimentation and modelling, the results for different rubber specimens and load cases were presented.

The primary aim of the research was to provide a novel method for modelling stress softening for localised stress concentration at discontinuities in rubber components and to implement optimised stress softening subroutines for rubber into commercial software codes. As a consequence other novel research is presented in respect of:

- i) A semi empirical formula for elastomer stress softening.
- ii) Recommendations for improving ICP techniques applied to rubber.
- iii) A determination of the likelihood of large flaws causing premature failures in rubber specimens.

**Keywords:** stress softening, elastomer, fatigue, stress concentration.

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Candidate

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## Abbreviations

3D	Three Dimensional
ACM	Acrylic Rubber
ASTM	American Standard for Testing of Materials
BS	British Standards
CCD	Charged Couple Device
CL	Chemiluminescence
CT	Computed Tomography
DIK	Deutsches Institut für Kautschuktechnologie
DIN	Deutsches Institut für Normung
DMA	Dynamic Mechanical Analysis
DTMA	Dynamic Thermal Mechanical Analysis
EPDM	Ethylene-Propylene-Diene Polymer
FE	Finite Element
FEA	Finite Element Analysis
ICP	Image Correlation Photogrammetry
ISO	International Organization for Standardization
JASO	Japanese Automotive Standards Organisation
MTS	Material Testing System
NBR	Acrylonitrile-Butadiene Rubber
NR	Natural Rubber
RBE2	Rigid Body Element Type 2
RSA	Rheometrics Solids Analyser
SAE	Society of Automotive Engineers
SEN	Single Edge Notched
SLS	Standard Linear Solid
TEM	Transmission Electron Microscope
UV	Ultraviolet
WLF	Williams-Landel-Ferry
XLD	Crosslink Density

## Nomenclature

$C_{ij}$	Mooney-Rivlin Coefficient
$E$	Young's modulus
$E'$	Elastic Storage modulus
$E''$	Elastic Loss modulus
$E^*$	Elastic Complex modulus
$G'$	Dynamic storage modulus in shear
$G''$	Dynamic loss modulus in shear
$I_1$	1 <sup>st</sup> Strain Invariant
$I_2$	2 <sup>nd</sup> Strain Invariant
$\tan \delta$	Loss factor
$\varepsilon$	Strain
$\sigma$	Stress
$\tau$	Relaxation time
$\eta$	Viscosity
$F$	Force
$K_{tu}$	Notch sensitivity factor
$K_{t\theta}$	Notch sensitivity angle
phr	Parts per hundred rubber
$t_{90}$	90% of time to complete curing
$T_g$	Glass transition temperature
$\lambda$	stretch or strain ratio
$W$	Strain energy density

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